(*iii*) $(+13) \times 13 = 13 \times 13 = 169$ (*iv*) $(-20) \times (-8) = 20 \times 8 = 160$

2. When the integers have unlike sign

The product of a positive integer and a negative integer (*i.e.*, when both the numbers have unlike signs), is always negative. We multiply a positive integer and a negative integer as whole numbers and put a minus sign (–) before the product.

Examples:

(*i*) $(-12) \times (+5) = -(12 \times 5) = -60$ (*ii*) $(+15) \times (-8) = -(15 \times 8) = -120$

(*iii*) $(-9) \times (+10) = -(9 \times 10) = -90$

Product of three or more negative integers

Look at the following examples:

(*i*) $(-6) \times (-5) = 30$

(*ii*) $(-6) \times (-5) \times (-4) = \{(-6) \times (-5)\} \times (-4) = 30 \times (-4) = -(30 \times 4) = -120$

(*iii*) $(-6) \times (-5) \times (-4) \times (-2) = \{(-6) \times (-5) \times (-4)\} \times (-2) = (-120) \times (-2) = 120 \times 2 = 240$

OR

$(-6) \times (-5) \times (-4) \times (-2) = \{(-6) \times (-5)\} \times \{(-4) \times (-2)\} = 30 \times 8 = 240$

(*iv*) $(-6) \times (-5) \times (-4) \times (-2) \times (-3) = \{(-6) \times (-5) \times (-4) \times (-2)\} \times (-3) = (30 \times 8) \times (-3)$
$= 240 \times (-3) = -(240 \times 3) = -720$

We observe that:

(*i*) If the number of negative integers in a product is even, then the product is a positive integer.

(*ii*) If the number of negative integers in a product is odd, then the product is a negative integer.

Properties of Multiplication of Integers

1. Closure property

The product of two integers is always an integer.

So, for any two integers a and b, $a \times b$ is an integer.

Examples:

(*i*) $5 \times 7 = 35$ (an integer) (*ii*) $9 \times (-4) = -36$ (an integer) (*iii*) $(-8) \times (-5) = 40$ (an integer)

2. Commutative property

Multiplication of integers is commutative.

For any two integers a and b, $a \times b = b \times a$

Examples:

(*i*) $9 \times (-5) = -(9 \times 5) = -45$ and $(-5) \times 9 = -(5 \times 9) = -45$

$\therefore \quad 9 \times (-5) = (-5) \times 9$

(*ii*) $(-7) \times (-10) = 7 \times 10 = 70$ and $(-10) \times (-7) = 10 \times 7 = 70$

$\therefore \quad (-7) \times (-10) = (-10) \times (-7)$

3. Associative property

Multiplication of integers is associative.

For any three integers a, b, c, $a \times (b \times c) = (a \times b) \times c$

Examples:

(*i*) $6 \times (5 \times 4) = 6 \times 20 = 120$ and $(6 \times 5) \times 4 = 30 \times 4 = 120$

$\therefore \quad 6 \times (5 \times 4) = (6 \times 5) \times 4$

(*ii*) $(-3) \times \{(-4) \times (-8)\} = (-3) \times 32 = -96$ and $\{(-3) \times (-4)\} \times (-8) = 12 \times (-8) = -(12 \times 8) = -96$

$\therefore \quad (-3) \times \{(-4) \times (-8)\} = \{(-3) \times (-4)\} \times (-8)$

4. (a) Multiplication of any integer by 0

Product of an integer and 0 is 0.

For any integer a, $a \times 0 = 0 \times a = 0$

Examples: (*i*) $5 \times 0 = 0$ (*ii*) $(-7) \times 0 = 0$

(b) Multiplicative identity

1 is the multiplicative identity for integers.

For any integer a, $a \times 1 = 1 \times a = a$

Examples: (*i*) $7 \times 1 = 7$ (*ii*) $(-9) \times 1 = -9$ (*iii*) $1 \times 8 = 8$ (*iv*) $1 \times (-5) = -5$

5. Distributive property

(*a*) *Multiplication of integers is distributive over their addition.*

For any three integers a, b, c, $a \times (b + c) = a \times b + a \times c$

Examples:

(*i*) $4 \times (7 + 9) = 4 \times 7 + 4 \times 9$

Here, $4 \times (7 + 9) = 4 \times 16 = 64$ and $4 \times 7 + 4 \times 9 = 28 + 36 = 64$

$\therefore \quad 4 \times (7 + 9) = 4 \times 7 + 4 \times 9$

(*ii*) $(-3) \times \{(-5) + (-8)\} = (-3) \times (-5) + (-3) \times (-8)$

Here $(-3) \times \{(-5) + (-8)\} = (-3) \times (-13) = 3 \times 13 = 39$ and $(-3) \times (-5) + (-3) \times (-8) = 15 + 24 = 39$

$\therefore (-3) \times \{(-5) + (-8)\} = (-3) \times (-5) + (-3) \times (-8)$

(*iii*) $9 \times \{(-4) + 5\} = 9 \times (-4) + 9 \times 5$

Here, $9 \times \{(-4) + 5\} = 9 \times (1) = 9$ and $9 \times (-4) + 9 \times 5 = -36 + 45 = 9$

$\therefore \quad 9 \times \{(-4) + 5\} = 9 \times (-4) + 9 \times 5.$

(*b*) *Multiplication of integers is distributive over their subtraction.*

For any three integers a, b, c, $a \times (b - c) = a \times b - a \times c$

Examples:

(*i*) $10 \times [6 - (-2)] = 10 \times 6 - 10 \times (-2)$

Here, $10 \times [6 - (-2)] = 10 \times (6 + 2) = 10 \times 8 = 80$ and $10 \times 6 - 10 \times (-2) = 60 + 20 = 80$

$\therefore 10 \times [6 - (-2)] = 10 \times 6 - 10 \times (-2)$

(*ii*) $(-15) \times [(-7) - (-1)] = (-15) \times (-7) - (-15) \times (-1)$

Here, $(-15) \times [(-7) - (-1)] = (-15) \times (-7 + 1) = (-15) \times (-6) = 90$

and $(-15) \times (-7) - (-15) \times (-1) = 105 - 15 = 90$

$\therefore \quad (-15) \times [(-7) - (-1)] = (-15) \times (-7) - (-15) \times (-1)$

Solved Examples

1. *Find each of the following products:*

(*i*) $3 \times (-1)$ (*ii*) $(-1) \times 225$

(*iii*) $(-21) \times (-30)$ (*iv*) $(-316) \times (-1)$

(*v*) $(-15) \times 0 \times (-18)$ (*vi*) $(-12) \times (-11) \times (10)$

(*vii*) $9 \times (-3) \times (-6)$ (*viii*) $(-18) \times (-5) \times (-4)$

(*ix*) $(-1) \times (-2) \times (-3) \times 4$ (*x*) $(-3) \times (-6) \times (-2) \times (-1)$

Sol. (*i*) $3 \times (-1) = -(3 \times 1) = -3$ (*ii*) $(-1) \times 225 = -(1 \times 225) = -225$

(*iii*) $(-21) \times (-30) = 21 \times 30 = 630$ (*iv*) $(-316) \times (-1) = 316 \times 1 = 316$

(*v*) $(-15) \times 0 \times (-18)$

$\because$ For any integer a, $a \times 0 = 0 \times a = 0$

$\therefore \quad (-15) \times 0 \times (-18) = \{(-15) \times 0\} \times (-18)$

$= 0 \times (-18) = 0$

(*vi*) $(-12) \times (-11) \times (10) = \{(-12) \times (-11)\} \times (10) = 132 \times 10 = 1320$

(*vii*) $9 \times (-3) \times (-6) = 9 \times \{(-3) \times (-6)\} = 9 \times 18 = 162$

(*viii*) $(-18) \times (-5) \times (-4) = \{(-18) \times (-5)\} \times (-4) = 90 \times (-4) = -360$

(*ix*) $(-1) \times (-2) \times (-3) \times 4 = \{(-1) \times (-2)\} \times (-3) \times 4 = 2 \times (-3) \times 4 = \{2 \times (-3)\} \times 4 = (-6) \times 4 = -24$

(*x*) $(-3) \times (-6) \times (-2) \times (-1) = \{(-3) \times (-6)\} \times \{(-2) \times (-1)\} = 18 \times 2 = 36.$

2. *Verify the following:*

(*i*) $18 \times [7 + (-3)] = (18 \times 7) + [18 \times (-3)]$

(*ii*) $(-21) \times [(-4) + (-6)] = [(-21) \times (-4)] + [(-21) \times (-6)]$

Sol. (*i*) **Given:** $18 \times [7 + (-3)] = (18 \times 7) + [18 \times (-3)]$

This is distributivity of multiplication over addition

L.H.S. $= 18 \times [7 + (-3)] = 18 \times [7 - 3]$

$= 18 \times 4 = 72$

R.H.S. $= (18 \times 7) + [18 \times (-3)]$

$= 126 + (-54) = 126 - 54 = 72$

So, $18 \times [7 + (-3)] = (18 \times 7) + [18 \times (-3)]$ is verified.

(*ii*) **Given:** $(-21) \times [(-4) + (-6)] = [(-21) \times (-4)] + [(-21) \times (-6)]$

This is distributivity of multiplication over addition

L.H.S. $= (-21) \times [(-4) + (-6)] = (-21) \times [-4 - 6]$

$= (-21) \times [-10] = 210$

R.H.S. $= [(-21) \times (-4)] + [(-21) \times (-6)] = [84] + [126] = 84 + 126 = 210$

Hence, L.H.S. = R.H.S. is verified

3. (*i*) *For any integer a, what is* $(-1) \times a$ *equal to?*

(*ii*) *Determine the integer whose product with* (-1) *is*

(*a*) -22 (*b*) 37 (*c*) 0

Sol. (*i*) $(-1) \times a = -a$

(*ii*) (*a*) $-22 = 22 \times (-1)$

So, product of 22 with (-1) is -22

(*b*) $37 = (-37) \times (-1)$

So, product of (-37) with (-1) is 37

(*c*) $0 = 0 \times (-1)$

So, product of 0 with (-1) is 0.

4. *Starting from* $(-1) \times 5$, *write various products showing some pattern to show* $(-1) \times (-1) = 1$.

Sol. We have $(-1) \times 5 = -5$

Now, multiplying (-1) with integers 4, 3, 2, 1, 0 and -1, we get

$(-1) \times 4 = -4$, $(-1) \times 3 = -3$, $(-1) \times 2 = -2$,

$(-1) \times 1 = -1$, $(-1) \times 0 = 0$, $(-1) \times (-1) = 1$

Hence, $(-1) \times (-1) = 1$ is shown by the above pattern.

5. *Find the product, using suitable properties:*

(*i*) $26 \times (-48) + (-48) \times (-36)$ (*ii*) $8 \times 53 \times (-125)$

(*iii*) $15 \times (-25) \times (-4) \times (-10)$ (*iv*) $(-41) \times 102$

(*v*) $625 \times (-35) + (-625) \times 65$ (*vi*) $7 \times (50 - 2)$

(*vii*) $(-17) \times (-29)$ (*viii*) $(-57) \times (-19) + 57$

Sol. (*i*) $26 \times (-48) + (-48) \times (-36) = 26 \times (-48) + (-36) \times (-48)$ [$\because$ $a \times b = b \times a$, commutativity]
$= [26 + (-36)] \times (-48)$ [Using distributivity property]
$= (-10) \times (-48) = 480$

(*ii*) $8 \times 53 \times (-125) = (8 \times 53) \times (-125)$
$= 424 \times (-125) = -(424 \times 125)$
$= -[424 \times (100 + 25)]$
$= -[424 \times 100 + 424 \times 25]$ [Distributivity of multiplication over addition]
$= -[42400 + 10600] = -53000$

(*iii*) $15 \times (-25) \times (-4) \times (-10)$
$= 15 \times (-25) \times (-10) \times (-4)$ [Multiplication is commutative for integers]
$= 15 \times (-10) \times (-25) \times (-4)$ [Commutativity of multiplication]
$= [15 \times (-10)] \times [(-25) \times (-4)]$
$= -(15 \times 10) \times (25 \times 4)$ [$\because$ $a \times (-b) = -(a \times b)$ and $(-a) \times (-b) = ab$]
$= -150 \times 100 = -15000$

(*iv*) $(-41) \times 102 = -(41 \times 102)$ [$\because$ $a \times (-b) = -(a \times b)$]
$= -[41 \times (100 + 2)]$
$= -[41 \times 100 + 41 \times 2]$ [Distributivity of multiplication over addition]
$= -[4100 + 82] = -4182$

(*v*) $625 \times (-35) + (-625) \times 65 = 625 \times (-35) + 625 \times (-65)$ [$\because$ $(-a) \times b = -(a \times b) = a \times (-b)$]
$= 625 \times [(-35) + (-65)]$ [Using distributive property of multiplication over addition]
$= 625 \times (-100) = -(625 \times 100)$ [$\because$ $a \times (-b) = -(a \times b)$]
$= -62500$

(*vi*) $7 \times (50 - 2) = (7 \times 50) - (7 \times 2)$ [Using distributive property of multiplication over subtraction]
$= 350 - 14 = 336$

(*vii*) $(-17) \times (-29) = 17 \times 29$ [$\because$ $(-a) \times (-b) = a \times b$]
$= 17 \times (30 - 1) = 17 \times 30 - 17 \times 1$ [Distributivity of multiplication over subtraction]
$= 510 - 17 = 493$

(*viii*) $(-57) \times (-19) + 57 = 57 \times 19 + 57 \times 1$ [$\because$ $(-a) \times (-b) = a \times b$ and $a \times 1 = a$]
$= 57 \times (19 + 1)$ [Distributivity of multiplication over addition]
$= 57 \times 20 = 1140$

6. *A certain freezing process requires that room temperature be lowered from* 40°C *at the rate of* 5°C *every hour. What will be the room temperature* 10 *hours after the process begins?*

Sol. Room temperature is to be lowered from 40°C at the rate of 5°C every hour.

$\therefore$ Room temperature 10 hours after the process begins
$= 40°C - (5°C \times 10) = 40°C - 50°C = -10°C$

7. *In a class test containing* 10 *questions,* 5 *marks are awarded for every correct answer and* (–2) *marks are awarded for every incorrect answer and* 0 *for questions not attempted.*

(*i*) *Mohan gets four correct and six incorrect answers, what is his score?*

(*ii*) *Reshma gets five correct answers and five incorrect answers, what is her score?*

(*iii*) *Heena gets two correct and five incorrect answers out of seven questions she attempts. What is her score?*

Sol. (*i*) Mohan gets four correct answers and six incorrect answers.

So, Mohan's score = $4 \times 5 + 6 \times (-2)$

$= 20 + (-12) = 20 - 12 = 8$ marks

(*ii*) Reshma gets five correct answers and five incorrect answers,

So, Reshma's score = $5 \times 5 + 5 \times (-2) = 25 - 10 = 15$ marks

(*iii*) Heena gets two correct and five incorrect answers. She did not attempt three questions.

So, her score = $2 \times 5 + 5 \times (-2) + 3 \times (0)$

$= 10 - 10 + 0 = 0$ marks.

8. *A cement company earns a profit of* ₹ 8 *per bag of white cement sold and a loss of* ₹ 5 *per bag of grey cement sold.*

(*i*) *The company sells* 3,000 *bags of white cement and* 5,000 *bags of grey cement in a month. What is its profit or loss?*

(*ii*) *What is the number of white cement bags it must sell to have neither profit nor loss, if the number of grey bags sold is* 6,400 *bags?*

Sol. A cement company earns a profit of ₹ 8 per bag of white cement sold and a loss of ₹ 5 per bag of grey cement sold.

(*i*) As company sells 3000 bags of white cement and 5,000 bags of grey cement in a month,

so its profit on 3,000 bags of white cement = ₹ $(3{,}000 \times 8)$ = ₹ 24,000

and its loss on 5,000 bags of grey cement = ₹ $(5{,}000 \times 5)$ = ₹ 25,000

As loss is more than profit,

So, loss of the company = ₹ 25,000 – ₹ 24,000

= ₹ 1,000

(*ii*) To have neither profit nor loss,

Number of white cement bags × profit per bag = Number of grey cement bags sold × loss per bag

$\therefore$ Number of white cement bags sold $\times$ 8 = 6400 $\times$ 5

$\Rightarrow$ Number of white cement bags = $\frac{6400 \times 5}{8} = 800 \times 5 = 4{,}000$

Hence, 4,000 white cement bags must be sold to have neither profit nor loss.

9. *Replace the blank with an integer to make it a true statement.*

(*i*) $(-3) \times$ ..... $= 27$ (*ii*) $5 \times$ ..... $= -35$

(*iii*) ..... $\times (-8) = -56$ (*iv*) ..... $\times (-12) = 132$

Sol. (*i*) $(-3) \times (\mathbf{-9}) = 27$ [Form $(-a) \times (-b) = (a \times b)$]

(*ii*) $5 \times (\mathbf{-7}) = -35$ [Form $a \times (-b) = -(a \times b)$]

(*iii*) $\mathbf{7} \times (-8) = -56$ [Form $a \times (-b) = -(a \times b)$]

(*iv*) $(\mathbf{-11}) \times (-12) = 132$ [Form $(-a) \times (-b) = (a \times b)$]

Exercise 1.3

1. Fill in the blanks:

(*i*) For any two positive integers a and b, $a \times (-b) = (-a) \times b =$ .....

(*ii*) For any two positive integers a and b, $(-a) \times (-b) =$ .....

(*iii*) Product of three negative integers is a ..... integer

(*iv*) Product of four negative integers is a ..... integer.

(*v*) If we multiply 4 positive integers and 3 negative integers, the sign of the product is .....

2. State true or false for each of the following statements. Justify your answers.
   (*i*) $a \times b$ is an integer, for all integers a and b.
   (*ii*) For any two integers a and b, $a \times b \neq b \times a$
   (*iii*) For any three integers a, b and c, $(a \times b) \times c = a \times (b \times c)$
   (*iv*) For any three integers a, b and c, $a \times (b + c) = a \times b + a \times c$
   (*v*) For any integer a, $a \times 1 = a$; 1 is the multiplicative identity for integers.
   (*vi*) For any integer a, $a \times 0 = 0 \times a = 0$
3. Find each of the following products:
   (*i*) -5×16 (*ii*) $24 \times (-3)$ (*iii*) 135×0 (*iv*) $(-3) \times (-3) \times (-3)$
   (*v*) $(-5) \times (-19) \times 0$ (*vi*) $(-4) \times (-7) \times (-5) \times 10$
4. Take $a = 5$, $b = -3$ and $c = 4$, verify that
   (*i*) $a \times b = b \times a$ (*ii*) $b \times c = c \times a$
   (*iii*) $a \times (b \times c) = (a \times b) \times c$ (*iv*) $a \times (b + c) = a \times b + a \times c$
   (*v*) $a \times (b - c) = a \times b - a \times c$
5. (*i*) Find the integer which when multiplied by -1 gives 99.
   (*ii*) Find the product using suitable property: $165 \times (-67) + (-165) \times 33$
   (*iii*) Find the product using suitable property: $35 + (-35) \times (-99)$
6. Every floor of a 20 storey building is 4 m high. A lift moves 2 m per second. How long will it take to move from 4th floor to 14th floor?
7. The temperature of a city was 28°C on Tuesday. The temperature rose by 1.5°C on everyday upto Friday and fell by 2.5°C on Saturday. What was the temperature of the city on Saturday?

DIVISION OF INTEGERS

Division is the inverse operation of multiplication

Since $7 \times 8 = 56.$

7 is contain into 56 eight times.

Thus $56 \div 7 = 8$

Similarly, $6 \times 5 = 30,$

So, $30 \div 6 = 5$

As $6 \times 5 = 5 \times 6$

So, $30 \div 5 = 6$

RULES FOR DIVIDING INTEGERS

1. **When integers have same sign:** The quotient of two integers, both positive or both negative (i.e. like signs), is always positive. We divide them as whole numbers.
   Examples:
   (*i*) $(+8) \div (+2) = \frac{8}{2} = 4$ (*ii*) $(-9) \div (-3) = \frac{9}{3} = 3$
   (*iii*) $(+12) \div 4 = \frac{12}{4} = 3$ (*iv*) $(-24) \div (-6) = \frac{24}{6} = 4$
2. **When integers have unlike sign:** When a negative integer is divided by a positive integer or when a positive integer is divided by a negative integer, the quotient is always negative. We divided them as whole number and then put a minus (–) sign before the quotient.
   Examples:
   (*i*) $63 \div (-9) = -7$ (*ii*) $(-25) \div (5) = -5$ (*iii*) $36 \div (-4) = -9$ (*iv*) $(-121) \div (11) = -11$

PROPERTIES OF DIVISION OF INTEGERS

1. *If a and b are integers, then $a \div b$ is not necessarily an integer.*

For example, $8 \div 3$ is not an integer, $24 \div 7$ is not an integer.

We see that *integers are not closed under division i.e.*, for two integers a and b, $a \div b$ may not be an integer.

2. *Division is not commutative for integers.*

For any two integers a and b, $a \div b \neq b \div a$

For example, $(-18) \div 3 = -6$

and $3 \div (-18) = -\frac{1}{6}$

$\therefore$ $(-18) \div 3 \neq 3 \div (-18)$

3. *Division is not associative for integers.*

If a, b, c are non-zero integers, then

$(a \div b) \div c \neq a \div (b \div c)$ unless $c = 1$

4. (*i*) For an integer a, different from 0, $a \div 1 = a$

For example, $-3 \div 1 = -3$, $15 \div 1 = 15$, etc.

(*ii*) If a is a non-zero integer, then

$0 \div a = 0$

For example, $0 \div (-5) = 0$, $0 \div 3 = 0$, etc.

(*iii*) If a is an integer, then $a \div 0$ is not meaningful. It is undefined.

Solved Examples

1. *Evaluate each of the following:*

(*i*) $(-30) \div 10$ (*ii*) $50 \div (-5)$ (*iii*) $(-36) \div (-9)$ (*iv*) $(-49) \div (49)$

(*v*) $13 \div [(-2) \div 1]$ (*vi*) $0 \div (-12)$ (*vii*) $(-31) \div [(-30) + (-1)]$

(*viii*) $[(-36) \div 12] \div 3$ (*ix*) $[(-6) + 5] \div [(-2) + 1]$

Sol. (*i*) $(-30) \div 10 = \frac{-30}{10} = -\frac{30}{10} = -3$ $\left[\because -a \div b = -\frac{a}{b} \text{ where } b \neq 0\right]$

(*ii*) $50 \div (-5) = \frac{50}{-5} = -\frac{50}{5} = -10$ $\left[\because a \div (-b) = -\frac{a}{b},\ b \neq 0\right]$

(*iii*) $(-36) \div (-9) = \frac{-36}{-9} = \frac{36}{9} = 4$ $\left[\because (-a) \div (-b) = \frac{a}{b},\ \text{where } b \neq 0.\right]$

(*iv*) $(-49) \div 49 = \frac{-49}{49} = -\frac{49}{49} = -1$

(*v*) $13 \div [(-2) + 1] = 13 \div (-1) = \frac{13}{-1} = -\frac{13}{1} = -13$ $\left[\because a \div (-b) = -\frac{a}{b},\ b \neq 0.\right]$

(*vi*) $0 \div (-12) = 0$ [$\because$ $0 \div a = 0$, where a is any integer]

(*vii*) $(-31) \div [(-30) + (-1)] = (-31) \div (-31) = \frac{(-31)}{(-31)} = 1$ [$\because$ For any integer a, $a \div a = 1$]

(*viii*) $[(-36) \div 12] \div 3 = \left[-\left(\frac{36}{12}\right)\right] \div 3 = \frac{(-3)}{3} = -\frac{3}{3} = -1$

(*ix*) $[(-6) + 5] = -1$ and $[(-2) + 1] = -1$

$\therefore$ $[(-6) + 5] \div [(-2) + 1] = (-1) \div (-1) = \frac{(-1)}{(-1)} = 1$ $\left[\because (-a) \div (-b) = \frac{a}{b} \text{ where } b \neq 0\right]$

2. *Verify that* $a \div (b + c) \neq (a \div b) + (a \div c)$ for each of the following values of a, b and c.

(*i*) $a = 12, b = -4, c = 2$ (*ii*) $a = (-10), b = 1, c = 1$

Sol. We have to verify that $a \div (b + c) \neq (a \div b) + (a \div c)$

(*i*) **Given:** $a = 12, b = -4, c = 2$.

$\therefore \quad a \div (b + c) = 12 \div (-4 + 2) = 12 \div (-2) = -\frac{12}{2} = -6$

$(a \div b) = 12 \div (-4) = \frac{12}{-4} = -\frac{12}{4} = -3$

and $\quad a \div c = 12 \div 2 = 6$

So, $\quad (a \div b) + (a \div c) = -3 + 6 = 3$

Hence, $\quad a \div (b + c) \neq (a \div b) + (a \div c)$ is verified

(*ii*) **Given:** $a = -10, b = 1, c = 1$

$\because \quad a \div (b + c) = (-10) \div (1 + 1) = (-10) \div 2 = -\frac{10}{2} = -5$

$a \div b = (-10) \div 1 = -10$

and $\quad a \div c = -10 \div 1 = -\frac{10}{1} = -10$

So, $\quad (a \div b) + (a \div c) = (-10 + (-10)) = -20.$

Hence, $\quad a \div (b + c) \neq (a \div b) + (a \div c)$

3. *Fill in the blanks.*

(*i*) $369 \div \ldots\ldots = 369$ (*ii*) $(-75) \div \ldots\ldots = -1$ (*iii*) $(-206) \div \ldots\ldots = 1$

(*iv*) $-87 \div \ldots\ldots = 87$ (*v*) $\ldots\ldots \div 1 = -87$ (*vi*) $\ldots\ldots \div 48 = -1$

(*vii*) $20 \div \ldots\ldots = -2$ (*viii*) $\ldots\ldots \div (4) = -3$

Sol. (*i*) $369 \div \mathbf{1} = 369$ (*ii*) $(-75) \div \mathbf{75} = -1$ (*iii*) $(-206) \div (\mathbf{-206}) = 1$

(*iv*) $-87 \div (\mathbf{-1}) = 87$ (*v*) $\mathbf{-87} \div 1 = -87$ (*vi*) $\mathbf{-48} \div 48 = -1$

(*vii*) $20 \div (\mathbf{-10}) = -2$ (*viii*) $\mathbf{-12} \div (4) = -3$

4. *Write five pairs of integers* (a, b) *such that* $a \div b = -3$. *One such pair is* $(6, -2)$ *because* $6 \div (-2) = -3$.

Sol. We have to write five pairs of integers (a, b) such that $a \div b = -3$.

(*i*) $[(-15), 5]$ (*ii*) $[18, (-6)]$ (*iii*) $[(-12), 4]$ (*iv*) $[24, (-8)]$ (*v*) $[(-27), 9]$

5. *The temperature at* 12 *noon was* 10°C *above zero. If it decreases at the rate of* 2°C *per hour until midnight, at what time would the temperature be* 8°C *below zero? What would be the temperature at mid-night?*

Sol. Difference between the temperature 10°C above zero and 8°C below zero = 10°C – (–8°C)

= 10°C + 8°C = 18°C

As rate of change in temperature = –2°C/hour

So, time for the difference of 18°C = 18 ÷ 2 = 9 hours

Difference in time from 12 noon and mid-night = 12 hours

Change in temperature in 12 hours = 12°C × (–2) = –24°C

$\therefore$ Temperature at mid-night = 10 + (–24) = 10 – 24 = –14°C

Hence, temperature 8°C below zero would be at 9 p.m. and temperature at mid-night = –14°C

6. *In a class test* (+3) *marks are given for every correct answer and* (–2) *marks are given for every incorrect answer and no marks for not attempting any question.*

(*i*) *Radhika scored* 20 *marks. If she has got* 12 *correct answers, how many questions has she attempted incorrectly?*

(*ii*) *Mohini scored* –5 *marks in the test though she has got* 7 *correct answers. How many questions has she attempted incorrectly?*

Sol. (*i*) (+3) marks are given for every correct answer and (–2) marks are given for every incorrect answer in a class test.

Total marks scored by Radhika = 20

Marks scored for 12 correct answers = $12 \times 3 = 36$.

Marks scored for incorrect answers = $20 - 36 = -16$

$\therefore$ Number of incorrect answers = $(-16) \div (-2) = 8$

Therefore, number of questions which were attempted incorrectly = 8

(*ii*) Marks scored by Mohini for 7 correct answers = $7 \times 3 = 21$ marks

Marks obtained by Mohini = –5

Marks obtained for incorrect answers = $-5 - 21 = -26$

$\therefore$ Number of incorrect answers = $\frac{-26}{-2} = 13$

Hence Mohini attempted 13 questions incorrectly.

7. *An elevator descends into a mine shaft at the rate of 6 m/min. If the descent starts from 10 m above the ground level, how long will it take to reach –350 m?*

Sol. Descent starts from 10 m above the ground level.

Descent has to move below the ground level 350 m

So, the total distance moved by the elevator = 350 m + 10 m = 360 m

As the elevator descends into a mine shaft at the rate of 6 m/minute

$\therefore$ Time taken to reach the mine shaft = $\frac{\text{Total distance}}{\text{Rate of descent}} = \frac{360}{6}$ = 60 minutes = 1 hour.

Exercise 1.4

1. State True or False for each of the following statements:

(*i*) For any two positive integers a and b, $a \div (-b) = (-a) \div b$, where $b \neq 0$

(*ii*) For any two positive integers a and b, $(-a) \div (-b) = a \div b$, where $b \neq 0$

(*iii*) Any integer divided by 1 gives the same integer *i.e.* for any integer a, $a \div 1 = a$.

(*iv*) $0 \div (-11) = 0$ or $0 \div 27 = 0$

(*v*) $(a \div b) \div c = a \div (b \div c)$

2. Fill in the blanks.

(*i*) $268 \div$ ..... $= 268$ (*ii*) $(-82) \div$ ..... $= -1$ (*iii*) ..... $\div (65) = 1$

(*iv*) ..... $\div 1 = -118$ (*v*) $60 \div$ ..... $= -6$ (*vi*) ..... $\div (7) = -3$

3. **Given:** (*i*) $a = 18, b = -3, c = 2$

(*ii*) $a = (-20), b = -1, c = -1$

Verify that $a \div (b \div c) \neq (a \div b) \div (a \div c)$ for each of the above stated values of a, b and c

4. Evaluate:

(*i*) $0 \div 75$ (*ii*) $(-125) \div (-5)$ (*iii*) $[(-8) + 7] \div [(-5) + 4]$

5. Find the value of x:

(*i*) $x \div (-14) = -7$ (*ii*) $x \div (18) = -3$

6. (*i*) Show that $(-100) \div (5)$ is same as $100 \div (-5)$

(*ii*) Show that $(-18) \div [6 \div (-1)] \neq [(-18) \div 6] \div (-1)$

(*iii*) Show that $\{(-72) \div (-9)\} \div \{(-20) \div (-5)\} = 2$

7. The product of two integers is 189. If one integer is –9, find the other.

8. The temperature at 6 pm of a town was 37°C. At mid-night the temperature was 31°C. If the rise in temperature every hour is same, find the rise in temperature per hour.

Miscellaneous Solved Examples

There are four options (in Q1 to Q15) out of which only one is correct. Choose the correct option.

1. *When the integers 10, 0, 5, –5, –7 are arranged in descending or ascending order, then find out which*

of the following integers always remains in the middle of the arrangement.

(*a*) 0 (*b*) 5 (*c*) 7 (*d*) –5.

Sol. When the integers 10, 0, 5, –5, –7 are arranged in ascending order, we have –7, –5, 0, 5, 10, here 0 is in the middle of the arrangement.

When the integers 10, 0, 5, –5, –7 are arranged in descending order, we have 10, 5, 0, –5, –7.

Here also, 0 is in the middle of the arrangement.

Clearly, 0 always remains in the middle of the arrangement.

Hence, (*a*) is the correct answer.

2. *Next three consecutive numbers in the patterns* 11, 8, 5, 2, ... *are*

(*a*) 0, –3, –6 (*b*) 1, –5, –8 (*c*) –2, –5,–8 (*d*) –1, –4, –7

Sol. Given numbers are 11, 8, 5, 2, ...

We observe that difference between two consecutive numbers is 3 as

$11 - 8 = 3, 8 - 5 = 3, 5 - 2 = 3$

$\therefore$ The next number will be $2 - 3 = -1$, the next two numbers are $-1 - 3 = -4, -4 - 3 = -7$

Thus, the next three consecutive numbers in the pattern 11, 8, 5, 2 ... are –1, –4, –7.

Hence, (*d*) is the correct answer.

3. *The next number in the pattern* –62, –37, –12, ... *is*

(*a*) 25 (*b*) 13 (*c*) 0 (*d*) –13

Sol. We have –62, –37, –12, ...

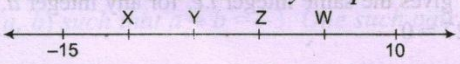
The difference between two consecutive numbers is $-37 - (-62) = -37 + 62 = 25$

and $-12 - (-37) = -12 + 37 = 25$

So, the next number in the pattern is $-12 + 25 = 13$

Hence, (*b*) is the correct answer.

4. *On the following number line value 'zero' is shown by the point*



(*a*) X (*b*) Y (*c*) Z (*d*) W

Sol. All points shown on the number line are equally spaced. Distance between two consecutive points = 5 units.

$\therefore$ $X = -15 + 5 = -10$, $Y = -10 + 5 = -5$, $Z = -5 + 5 = 0$

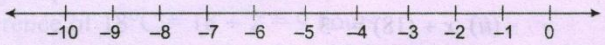
Clearly, 'zero' (0) is shown by the point Z.

Hence, (*c*) is the correct answer.

5. *The value of* $5 \div (-1)$ *does not lie between*

(*a*) 0 and –10 (*b*) 0 and 10 (*c*) –4 and –15 (*d*) –6 and 6

Sol. $5 \div (-1)$ equals to –5



Clearly –5 lies between (0 and –10), (–4 and –15) and (–6 and 6).

But –5 does not lie between (0 and 10).

Hence, (*b*) is the correct answer.

6. $(-11) \times 7$ *is not equal to*

(*a*) $11 \times (-7)$ (*b*) $-(11 \times 7)$ (*c*) $(-11) \times (-7)$ (*d*) $7 \times (-11)$

Sol. We know that product of a positive integer and a negative integer is a negative and product of two positive or negative integers is a positive integer.

$\therefore$ $(-11) \times 7 = -77$, $11 \times (-7) = -77$, $-(11 \times 7) = -77$

$(-11) \times (-7) = 77$ and $7 \times (-11) = -77$

Clearly, $(-11) \times 7$ is not equal to $(-11) \times (-7) = 77$

Hence, (*c*) is the correct answer.

7. $(-10) \times (-5) + (-7)$ *is equal to*

(*a*) -57 (*b*) 57 (*c*) -43 (*d*) 43

Sol. We have $(-10) \times (-5) + (-7)$

Now, $(-10) \times (-5) = (10 \times 5) = 50$

$\therefore \quad (-10) \times (-5) + (-7) = 50 + (-7)$

$= 50 - 7 = 43$

Hence, (*d*) is the correct answer.

8. *Which of the following is not the additive inverse of a?*

(*a*) $-(-a)$ (*b*) $a \times (-1)$ (*c*) $-a$ (*d*) $a \div (-1)$

Sol. $-(-a) = a$, $a \times (-1) = -a$

$-a = -a$ and $a \div (-1) = \dfrac{a}{-1} = -a$

Additive inverse of a is $(-a)$.

So, $-(-a) = a$ is not the additive inverse of a.

Hence, (*a*) is the correct answer.

9. *Which of the following is the multiplicative identity for an integer a?*

(*a*) a (*b*) 1 (*c*) 0 (*d*) -1

Sol. Multiplicative identity for an integer a is 1 because $a \times 1 = a$. If we multiply any integer by 1, it gives us the same integer.

Hence, (*b*) is the correct answer.

10. $(-25) \times [6 + 4]$ *is not same as*

(*a*) $(-25) \times 10$ (*b*) $(-25) \times 6 + (-25) \times 4$

(*c*) $(-25) \times 6 \times 4$ (*d*) -250

Sol. $(-25) \times [6 + 4] = (-25) \times 10 = -250$

Now, we evaluate the given options.

$(-25) \times 10 = -250$

Again by distributivity of multiplication over addition

$(-25) \times [6 + 4] = (-25) \times 6 + (-25) \times 4$

$= -150 - 100 = -250$

$(-25) \times 6 \times 4 = (-25) \times 24 = -600$

and $-250 = -250$

Clearly, $(-25) \times [6 + 4]$ is not the same as $(-25) \times 6 \times 4$.

Hence, (*c*) is the correct answer.

11. $(-43) \times (-99) + 43$ *is equal to*

(*a*) 4300 (*b*) -4300 (*c*) 4257 (*d*) -4214

Sol. $(-43) \times (-99) + 43 = 43 \times 99 + 43$ [$\because$ $(-a) \times (-b) = ab$, where a and b are positive integers]

$= 43(99 + 1)$

$= 43 \times 100 = 4300.$

Hence, (*a*) is the correct answer.

12. *Which of the following does not represent an integer?*

(*a*) $0 \div (-7)$ (*b*) $20 \div (-4)$ (*c*) $(-9) \div 3$ (*d*) $(-12) \div 5$

Sol. $0 \div (-7) = \dfrac{0}{-7} = 0$ which is an integer

$20 \div (-4) = \dfrac{20}{-4} = -\dfrac{20}{4} = -5$, which is an integer

$(-9) \div 3 = \dfrac{-9}{3} = -\dfrac{9}{3} = -3$, which is an integer.

$(-12) \div 5 = \dfrac{-12}{5} = -\dfrac{12}{5}$, which is a fraction.

Hence, (*d*) is the correct answer.

13. *Which of the following shows the maximum rise in temperature ?*

(*a*) 23° to 32° (*b*) −10° to +1° (*c*) −18° to −11° (*d*) −5° to 5°

Sol. (*a*) Rise in temperature = 32° − 23° = 9°

(*b*) Rise in temperature = 1° − (−10°) = 1° + 10° = 11°

(*c*) Rise in temperature = −11° − (−18°) = −11° + 18° = 7°

(*d*) 5° − (−5°) = 5° + 5° = 10°

Hence, (*b*) −10° to +1° shows the maximum rise in temperature

14. *If a and b are two integers, then which of the following may not be an integer ?*

(*a*) $a + b$ (*b*) $a - b$ (*c*) $a \times b$ (*d*) $a \div b$

Sol. We know that integers are closed under addition, subtraction and multiplication but integers are not closed under division.

Hence, (*d*) $a \div b$ may not be an integer.

15. *For a non-zero integer a, which of the following is not defined ?*

(*a*) $a \div 0$ (*b*) $0 \div a$ (*c*) $a \times b$ (*d*) $a \div b$

Sol. For any integer a, $a \div 0$ is not defined. Any integer divided by zero is meaningless.

Hence, (a) $a \div 0$ is not defined.

16. *In the following questions, fill in the blanks to make the statements true.*

(*i*) $(-a) + b = b +$ *Additive inverse of* .....

(*ii*) ..... $\div (-10) = 0$

(*iii*) $(-157) \times (-19) + 157 =$ .....

(*iv*) *If x, y and z are integers, then* $(x +$ .....$) + z =$ ..... $+ (y +$ .....$)$

(*v*) $(-43) +$ ..... $= -43$ (*vi*) $(-9) \times 20 =$ .....

(*vii*) $(-23) \times (42) = (-42) \times$ ..... (*viii*) ..... $\times (-93) = 93$

(*ix*) *If we multiply five positive integers and one negative integer, then the resulting integer is* .....

(*x*) *When we divide a negative integer by a positive integer, we divide them as whole numbers and put a* ..... *sign before quotient.*

(*xi*) *When* −16 *is divided by* ..... *the quotient is* 4.

(*xii*) $(-100) \div (-10) =$ ..... (*xiii*) $(-225) \div 5 =$ .....

(*xiv*) ..... $\div (-1) = 75$ (*xv*) $(-95) \div$ ..... $= 95$

(*xvi*) $(-28) \div (-28) =$ .....

Sol. (*i*) $(-a) + b = b +$ Additive inverse of a because $-a$ is the additive inverse of a.

(*ii*) We know that for any integer a, $0 \div a = 0$ for $a \neq 0$.

Hence, $\mathbf{0} \div (-10) = 0$

(*iii*) $(-157)\,(-19) + 157 = (157)\,(19) + 157$ [$\because$ $(-a)\,(-b) = ab$, where a and b are positive integers]

$= 157(19 + 1) = 157 \times 20 = \mathbf{3140}$

(*iv*) For any integers a, b, c we have

$a + (b + c) = (a + b) + c$ [Addition is associative for integers]

$\therefore$ If x, y and z are integers, then

$(x + y) + z = \boldsymbol{x} + (y + \boldsymbol{z})$

(*v*) We know that 0 is an additive identity for integers *i.e.* for any integer a

$$a + 0 = a = 0 + a$$

So, $(-43) + \mathbf{0} = (-43)$

(*vi*) We know that product of a positive integer and a negative integer is a negative integer.

$\therefore$ $(-9) \times 20 = \mathbf{-180}$

(*vii*) As product of a positive integer and a negative integer is a negative integer. Also multiplication is commutative for integers *i.e.* $a \times b = b \times a$

$\therefore$ $(-23) \times 42 = (-42) \times \mathbf{23}$

(*viii*) As product of two negative integers is positive, so here product is 93 and one negative number is –93

So, $\mathbf{(-1)} \times (-93) = 93$

(*ix*) Multiplication of five positive integers is positive and when this positive integer is multiplied by a negative integer, then the resulting integer is **negative**.

(*x*) When we divide a negative integer by a positive integer or a positive integer by a negative integer, we divide them as whole numbers and put a **negative** sign before quotient.

(*xi*) Let when –16 is divided by x, the quotient is 4.

Then $-16 \div x = 4 \Rightarrow x = -16 \div 4 = -4$

Thus, when – 16 is divided by **–4**, the quotient is 4.

(*xii*) $(-100) \div (-10) = 100 \div 10 = \mathbf{10}$ [$\because$ For any two positive integers $(-a) \div (-b) = a \div b$, where $b \neq 0$]

(*xiii*) $(-225) \div 5 = -\dfrac{225}{5} = \mathbf{-45}$ $\left[\because\ (-a) \div b = -\dfrac{a}{b}\right.$ where a and b are positive integers and $\left.-\dfrac{a}{b}\right.$ is an integer$\left.\right]$

(*xiv*) We have ..... $\div (-1) = 75$

So, $\mathbf{-75} \div (-1) = 75$ [$\because$ Division of a negative integer by another negative integer gives a positive integer as quotient]

(*xv*) Given: $(-95) \div$ ..... $= 95$

So, $(-95) \div \mathbf{(-1)} = 95$ [Same reason as in part (*xiv*)]

(*xvi*) Given: $(-28) \div (-28) =$ .....

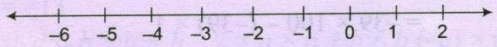
So, $(-28) \div (-28) = \mathbf{1}$ [Same reason as in part (*xiv*) and (*xv*)]

17. *Write a pair of integers whose product is* –12 *and there lies seven integers between them* (*excluding the given integers.*)

Sol. Consider a pair of integers (–6, 2)

Product of –6 and 2 = $-6 \times 2 = -12$.

Consider the following number line.

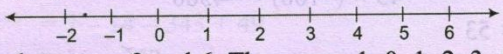


There are 7 integers lying between –6 and 2 which are –5, –4, –3, –2, –1, 0, 1

Again consider a pair of integers –2 and 6

Their product $= -2 \times 6 = -12$

Observe the following number line.



There are 7 integers lying between –2 and 6. These are –1, 0, 1, 2, 3, 4, 5

Hence, for pair of integers (–6, 2) or (2, –6), product is 12 and seven integers lie between them.

18. *From given integers in column I match each integer of column II so that their product lies between* –19 *and* –6.

Column I	Column II
–5	1
6	–1
–7	3
8	–2

Sol. Column I and column II are matched below in such a way that their product lies between –19 and –6

Column I	Column II
–5	3
6	–2
–7	1
8	–1

Products are: $-5 \times 3 = -15$, $6 \times (-2) = -12$, $-7 \times 1 = -7$ and $8 \times (-1) = -8$

–15, –12, –7 and –8 all lie between –19 and –6

19. *Write a pair of integers whose product is* –36 *and whose difference is* 15.

Sol. Consider a pair of integers (12, –3)

Product of 12 and –3 is $12 \times (-3) = -(12 \times 3) = -36$

and their difference $= 12 - (-3) = 12 + 3 = 15$

Hence, for pair of integers (12, –3), product of integers is –36 and their difference is 15.

20. (*i*) *Write two integers such that one is smaller than* –11 *and other is greater than* –11 *but their difference is* –11.

(*ii*) *Write two integers whose product is smaller than both the integers.*

(*iii*) *Write two integers whose product is greater than both the integers.*

Sol. (*i*) Consider the two integers –20 and –9.

Here, $-20 < -11$ and $-9 > -11$.

Their difference $= -20 - (-9) = -20 + 9 = -11$

(*ii*) Let one integer be –2 and other integer 4.

Their product $= -2 \times 4 = -(2 \times 4) = -8$

We see that product $= -8$ such that $-8 < -2$ and $-8 < 4$

(*iii*) Let one integer be 5 and other integer 6.

Their product $= 5 \times 6 = 30$

Here, product = 30 such that $30 > 5$ and $30 > 6$

21. *Evaluate the following by using distributive property.*

(*i*) -39×99 (*ii*) $(-85) \times 43 + 43 \times (-15)$

(*iii*) $53 \times (-9) - (-109) \times 53$ (*iv*) $68 \times (-17) + (-68) \times 3$

Sol. (*i*) $-39 \times 99 = -39 \times (100 - 1)$

$= -39 \times 100 - (-39) \times 1$

$= -3900 + 39 = -3861$

(*ii*) $(-85) \times 43 + 43 \times (-15) = 43 \times (-85) + 43 \times (-15)$ [Using commutativity of multiplication]

$= 43 \times (-85 - 15)$ [Taking out 43 common and using distributive property of multiplication]

$= 43 \times (-100) = -4300$

(*iii*) $53 \times (-9) - (-109) \times 53$

$= 53 \times (-9) - 53 \times (-109)$ [Commutative property of multiplication]

$= 53 [-9 - (-109)]$ [Taking out 53 common using distributive property of multiplication]

$= 53\ (-9 + 109) = 53\ (100)$

$= 5300$

(*iv*) $68 \times (-17) + (-68) \times 3$

$= 68 \times (-17) + 68 \times (-3)$ [$\because (-a) \times b = a \times (-b) = -ab$]

$= 68\ [-17 + (-3)]$ [Taking out 68 common using distributive property of multiplication]

$= 68 \times (-20)$

$= -1360$

22. *If $*$ is an operation such that for integers a and b we have $a * b = a \times b + (a \times a + b \times b)$, then find*

(*i*) $(-3) * (-5)$ (*ii*) $(-6) * 2$

Sol. **Given:** $a * b = a \times b + (a \times a + b \times b)$

(*i*) $(-3) * (-5) = (-3) \times (-5) + \{(-3) \times (-3) + (-5) \times (-5)\}$

$= (3 \times 5) + \{(3 \times 3) + (5 \times 5)\}$

$= 15 + (9 + 25) = 15 + 34 = 49$

(*ii*) $(-6) * 2 = -(6 \times 2) + \{(-6) \times (-6) + 2 \times 2\}$

$= -12 + (6 \times 6) + (2 \times 2)$

$= -12 + 36 + 4 = -12 + 40 = 28$

23. *If Δ is an operation such that for integers a and b we have*

$a \Delta b = a \times b - 2 \times a \times b + b \times b(-a) \times b + b \times b$

then find (*i*) $4 \Delta (-3)$ (*ii*) $(-7) \Delta (-1)$

Also show that $4 \Delta (-3) \neq (-3) \Delta 4$ and $(-7) \Delta (-1) \neq (-1) \Delta (-7)$

Sol. (*i*) $a \Delta b = a \times b - 2 \times a \times b + b \times b(-a) \times b + b \times b$

Taking, $a = 4$ and $b = (-3)$, we have

$4 \Delta (-3) = 4 \times (-3) - 2 \times 4 \times (-3) + (-3) \times (-3)(-4) \times (-3) + (-3) \times (-3)$

$= -(4 \times 3) - 2\{-(4 \times 3)\} + (3 \times 3 \times 4 \times 3) + (3 \times 3)$

$= -12 + 24 + 108 + 9 = -12 + 141 = 129$

Again $a \Delta b = a \times b - 2 \times a \times b + b \times b(-a) \times b + b \times b$

Taking $a = -3$ and $b = 4$, we get

$-3 \Delta 4 = (-3) \times 4 - 2 \times (-3) \times 4 + 4 \times 4\{-(-3)\} \times 4 + 4 \times 4$

$= -(3 \times 4) - \{-(2 \times 3 \times 4)\} + 16 \times 3 \times 4 + 16$

$= -12 + 24 + 192 + 16 = 12 + 192 + 16 = 220$

Clearly, $4 \Delta (-3) \neq (-3) \Delta 4$

(*ii*) $a \Delta b = a \times b - 2 \times a \times b + b \times b\ (-a) \times b + b \times b$

Taking $a = -7$ and $b = -1$, we have

$(-7) \Delta (-1) = (-7) \times (-1) - 2 \times (-7) \times (-1) + (-1) \times (-1)\ \{-(-7)\} \times (-1) + (-1) \times (-1)$

$= (7 \times 1) - (2 \times 7 \times 1) + (1 \times 1)\ \{-(7 \times 1)\} + (1 \times 1)$

$= 7 - 14 - 7 + 1 = -14 + 1$

$= -13$

Again, $a \Delta b = a \times b - 2 \times a \times b + b \times b\ (-a) \times b + b \times b$

Taking $a = -1$ and $b = -7$, we have

$(-1) \Delta (-7) = (-1) \times (-7) - 2 \times (-1) \times (-7) + (-7) \times (-7)\ \{-(-1)\} \times (-7) + (-7) \times (-7)$

$= (1 \times 7) - (2 \times 1 \times 7) + (7 \times 7)\ \{-(1 \times 7)\} + (7 \times 7)$

$= 7 - 14 + 49\ (-7) + 49$

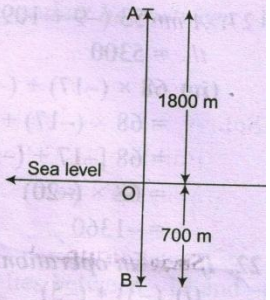
$= 7 - 14 - 343 + 49$

$= 56 - 357$

$= -301$

Clearly, $(-7) \Delta (-1) \neq (-1) \Delta (-7)$

24. *Height of a place A is* 1800 *m above sea level. Another place B is* 700 *m below sea level.*
What is the difference between the levels of these two places?



Sol. Height of a place A is 1800 m above sea level *i.e.* OA = 1800 m
Place B is 700 m below sea level
Distance between sea level and place B = 700 m *i.e.* OB = 700 m
Hence, required distance between levels of two places A and B
= 1800 m + 700 m = 2500 m

25. *The given table shows the freezing points in °F of different gases at sea level. Convert each of thes into °C to the nearest integral value using the relation and complete the table*

$$C = \frac{5}{9}(F - 32)$$

Gas	Freezing point at sea level (°F)	Freezing point at sea level (°C)
Hydrogen	–435	
Krypton	–251	
Oxygen	–369	
Helium	–458	
Argon	–309	

Sol. Given relation is $C = \frac{5}{9}(F - 32)$

For Hydrogen, $F = -435$

So, $C = \frac{5}{9}(-435 - 32) = \frac{5}{9} \times (-467) = \frac{1}{9}(-2335) = -259.44°C$

For Krypton, $F = -251$

So, $C = \frac{5}{9}(-251 - 32) = \frac{5}{9}(-283) = \frac{1}{9}(-1415) = -157.22°C$

For Oxygen, $F = -369$

So, $C = \frac{5}{9}(-369 - 32) = \frac{5}{9}(-401) = \frac{1}{9}(-2005) = -222.7°C$

For Helium, $F = -458$

So, $C = \frac{5}{9}(-458 - 32) = \frac{5}{9}(-490) = \frac{1}{9}(-2450) = -272.22°C$

For Argon, $F = -309$

So, $C = \frac{5}{9}(-309 - 32) = \frac{5}{9}(-341) = \frac{1}{9}(-1705) = -189.44°C$

26. *A green grocer had a profit of* ₹ 47 *on Monday, a loss of* ₹ 12 *on Tuesday and loss of* ₹ 8 *on Wednesday. Find his net profit or loss in* 3 *days.*

Sol. According to the question, we have
Profit of green grocer on Monday = ₹ 47
Loss of green grocer on Tuesday = ₹ 12
Loss of green grocer on Wednesday = ₹ 8
Total loss = ₹ 12 + ₹ 8 = ₹ 20
∴ Net profit = Total profit – Total loss = ₹ 47 – ₹ 20 = ₹ 27

27. *A multistorey building has 25 floors above the ground level each of height 5 m. It also has 3 floors in the basement each of height 5 m. A lift in building moves at a rate of 1 m/s. If a man starts from 50 m above the ground, how long will it take him to reach at 2nd floor of basement?*

Sol. A multistory building has 25 floors above the ground each of height 5 m.

Building has 3 floors in the basement each of height 5 m.

Man starts from 50 m above the ground and the distance covered by him below the ground upto 2nd floor

$$= 50 \text{ m} + (2 \times 5) \text{ m} = 50 \text{ m} + 10 \text{ m} = 60 \text{ m}$$

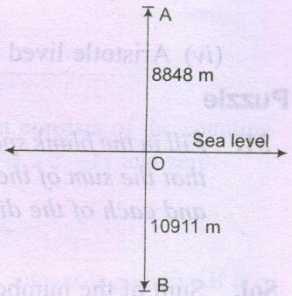
Speed of lift = 1 m/s

Now, $\text{Distance} = \text{Speed} \times \text{Time}$

So, time taken by the man to reach 2nd floor of the basement $= \dfrac{\text{Distance}}{\text{Speed}}$

$= \dfrac{60}{1} = 60$ seconds = 1 minute

28. *The highest point measured above sea level is the summit of Mt. Everest which is 8,848 m above sea level and the lowest point is Challenger Deep at the bottom of Mariana Trench which is 10911 m below sea level. What is the vertical distance between these two points?*



Sol. Let A be the highest point measured above the sea level which is the summit of Mt. Everest and OA = 8848 m

and B be the lowest point is the Challenger Deep at the bottom of Mariana Trench which is 10911 m below sea level *i.e.*, OB = 10911 m

$\therefore$ Vertical distance between the points A and B

$= OA + OB = 8848 + 10911 = 19759$ m

Application based Questions

29. ***Science Application:*** *An atom consists of charged particles called electrons and protons. Each proton has a charge of +1 and each electron has a charge of –1.*

Remember that number of electrons is equal to number of protons, while answering these questions.

(*i*) *What is the charge on an atom?*

(*ii*) *What will be the charge on an atom if it loses an electron?*

(*iii*) *What will be the charge on an atom if it gains an electron?*

Sol. (*i*) Let a be the number of electrons and as the number of electrons is equal to the number of protons, so number of protons is also equal to a.

As each proton has charge of +1 and each electron has a charge of –1

So total charge on a protons $= a \times (+1) = a$

and total charge on a electrons $= a \times (-1) = -a$

Hence, the total charge on an atom $= a + (-a) = a - a = 0$

(*ii*) If an atom loses an electron, then there will $(a - 1)$ electrons left and a protons.

Then, total charge on the atom $= (a - 1) \times (-1) + a \times 1 = 1 - a + a = 1$

(*iii*) If an atom gains an electron, then there will be $(a + 1)$ electrons and a protons.

Then, total charge on the atom $= (a + 1) \times (-1) + a \times 1 = -a - 1 + a = (-1)$

30. ***Social Studies Application:*** *Remembering that 1 AD came immediately after 1 BC, while solving these problems take 1 BC as –1 and 1 AD as +1.*

(*i*) *The Greco-Roman era, when Greece and Rome ruled Egypt, started in the year 330 BC and ended in the year 395 A.D. How long did the era last?*

(*ii*) *Bhaskaracharya was born in the year 1114 AD and died in the year 1185 AD. What was his age when he died?*

(*iii*) *Turks ruled Egypt in the year* 1517 *AD and Queen Nefertis ruled Egypt about* 2900 *years before the Turks ruled. In what year did she rule?*

(*iv*) *Greek mathematician Archimedes lived between* 287 *BC and* 212 *BC and Aristotle lived between* 380 *BC and* 322 *BC. Who lived during earlier period?*

Sol. (*i*) The period of Greeco-Roman era = End year – Start year
= 395 AD – (330 BC)
= 395(+1) – (330)(–1)
= 395 + 330 = 725 years.

(*ii*) Age of Bhaskaracharya when he died = (1185 AD) – (1114 AD)
= (1185)(+1) – (1114) (+1)
= 1185 – 1114 = 71 years.

(*iii*) Ruling period of Queen Nefertis = Year in which Turks ruled – 2900 years
= (1517 AD) – 2900 = 1517 (+1) – 2900 = 1517 – 2900
= –1383 = 1383(–1) = 1383 BC.

(*iv*) Aristotle lived (380 BC – 322 BC) earlier than Archimedes (287 BC – 212 BC)

Puzzle

31. *Fill in the blank space of the following magic square, so that the sum of the numbers in each row, each column and each of the diagonals is* –6.

–1		
3	–2	
	5	

Sol. Sum of the numbers in each row = –6
Sum of the numbers in each column = –6
Sum of the numbers in each diagonal = –6

–1	–9	4
3	–2	–7
–8	5	–3

Riddle

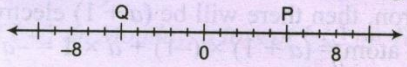
32. *Solve the following riddle*
Minus of minus six
Minus minus-minus-seven
What do you get if this added to
Minus minus-seven again?

Sol. –(–6) – {–(–7)} + {–(–7)}
= 6 + (–7) – (–7)
= 6 – 7 + 7 = 6

Exercise 1.5 (Miscellaneous)

There are four options in (Q1 to Q3) out of which only one is correct. Choose the correct option.

1. By observing the number line given below find which of the following statements is not true.



(*a*) Q is greater than –8 (*b*) P is greater than 0
(*c*) Q is greater than P (*d*) Q is smaller than 0

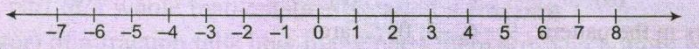
2. Next three consecutive numbers in the pattern 15, 12, 9, 6, ... are
(*a*) 3, 4, 5 (*b*) 3, 0, –3 (*c*) –2, –6, –7 (*d*) 2, 0, 3

3. Which of the following is not an integer?
(*a*) 0 ÷ 5 (*b*) (–4) × (–1) (*c*) (35) ÷ (–7) (*d*) (–17) ÷ (–12)

4. Fill in the blanks:
   (*i*) If we multiply 3 negative integers and 4 positive integers, then the resulting integer is ..... .
   (*ii*) $15 \times (-99) =$ ..... $\times (-100 +$ .....$) = 15 \times$ ..... $+ 15 \times$ .....
   (*iii*) When -25 is divided by ..... the quotient is 5.
5. Write true or false for each of the following statements.
   (*i*) For any three integers a, b and c, $a \times (b - c) = a \times b - a \times c$
   (*ii*) For any two positive integers a and b, $a \div (-b) = (-a) \div b$, where $b \neq 0$
   (*iii*) For any integer a, $a \div 1 = a$
   (*iv*) For any two positive integers a and b, $a \times (-b) = (-a) \times b = -(a \times b)$
   (*v*) $\{(-36) \div 6\} \div 3 = -36 \div [6 \div 3]$
   (*vi*) $(-25) \times [15 + (-5)] = [(-25) \times 15] + [(-25) \times (-5)]$
   (*vii*) The successor of $0 \times (-21)$ is $1 \times (-21)$.
6. Write a pair of integers whose product is -39 and whose difference is 16.
7. Write five pairs of integers (a, b) such that $a \div b = -2$. One such pair is $(8, -4)$ because $8 \div (-4) = (-2)$

Points to Remember

1. Integers are the set of numbers {...., –5, –4, –3, –2, –1, 0, 1, 2, 3, 4, 5, ...} that consists of the counting numbers, {1, 2, 3, 4, 5, ...}. The negative of the counting numbers, {...., –5, –4, –3, –2, –1} and zero.
2. Integers can be shown on a number line.



3. An integer is positive if it is greater than zero. They are on the right side of 0. An integer is negative if it is less than zero. They are on the left side of 0.
4. **Properties of Integers:**
   (*i*) **Closure property under addition and subtraction of integers:** For two integers a and b, $a + b$ is an integer and $a - b$ is an integer.
   (*ii*) **Commutative property of addition:** Adding integers will have the same result regardless of the arrangement of the integers. For example, $5 + 3 = 3 + 5$
   For two integers a and b, $a + b = b + a$
   So, addition is commutative for all integers.
   (*iii*) **Addition is associative for integers:** Adding integers will have the same result regardless of the grouping. For integers a, b and c
   $(a + b) + c = a + (b + c)$
   ***Note*:** Subtraction is neither commutative nor associative.

Identity element: 0 is the identity under addition. For any integer, $a + 0 = 0 + a = a$

Properties of Integers under multiplication and division:

(*i*) **Closure property:** The set of integers is closed **under** the operation of multiplication because product of any two integers is always another integer. Therefore, for integers a and b, $a \times b$ is also an integer.

***Note*:** Division of integers does not follow the closure property i.e. the quotient of any two integers a and b, may or may not be an integer.

(*ii*) **Commutative property:** Multiplying integers will have the same result regardless of the arrangement of the integers. For any integers a and b
$a \times b = b \times a$.
For example , $5 \times 7 = 7 \times 5$

(*iii*) **Associative property:** Multiplying integers will have the same result regardless of the grouping. That is, the product will not change even if the integers are grouped differently. For any three integers a, b and c, we have
$(a \times b) \times c = a \times (b \times c)$

Note: Division is neither commutative nor associative.

(*iv*) **Identity element:** If any integer is multiplied by 1, the product is that original number. So, for any integer a, $1 \times a = a \times 1 = a$, the number 1 is called the identity element for multiplication.

(*v*) **Distributive property:** The distributive property lets you multiply a sum by multiplying each addend separately and then add the products.

For any three integers a, b and c

$a \times (b + c) = a \times b + a \times c$

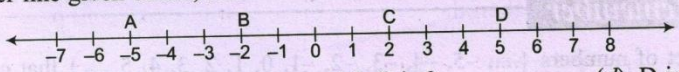
Distributive property involves the addition of integers being multiplied by another integer. We say that multiplication distributes over addition.

(*vi*) For any integer a, $a \div 0$ is not defined and $a \div 1 = a$.

CHAPTER TEST

I. Choose the correct answers

1. Observe the number line given below, state which of the following statements is true.



(*a*) B is –3 (*b*) A is –6 (*c*) C is 2 (*d*) D is 7

2. On the number line, the value –5 + 9 lies to the left of

(*a*) 0 (*b*) 5 (*c*) –9 (*d*) –2

3. Next three numbers in the pattern –9, –6, –3, 0, ... are

(*a*) –12, –15, –18 (*b*) –5, –2, 1 (*c*) 3, 6, 9 (*d*) 4, 7, 10

4. Which of the following is the not the additive inverse of 7?

(*a*) –(–7) (*b*) –7 (*c*) 7 × (–1) (*d*) 7 ÷ (–1)

5. Which of the following does not represent integer?

(*a*) 30 ÷ (–6) (*b*) (–15) ÷ 5 (*c*) 0 ÷ (–2) (*d*) (–18) ÷ 7

II. Fill in the blanks:

6. (*i*) 15 × (–1) = ... (*ii*) –8 – (–9) = ...

(*iii*) 6 + (–13) = ... (*iv*) (–5) × (–5) + (–6) × 3 = ...

III. State true (T) or false (F) for each of the following statements.

7. (*i*) –6 and –3 are less than –1. (*ii*) 8 – (–8) = 16

(*iii*) (–90) ÷ 5 = –16 (*iv*) (–7) × (–3) × (–5) × 2 = –120

8. (*i*) What is the difference between –7 and –10?

(*ii*) Give an example to show that the subtraction of two integers is not commutative.

(*iii*) Verify (–11) × {(–5) + (–7)} = {(–11) × (–5)} + {(–11) × (–7)}

(*iv*) Evaluate {(–40) ÷ 5} ÷ 2

(*v*) Write down the pair of integers whose sum is 0.

9. (*i*) What is (–8) × 1 equal to?

(*ii*) Which property is used here, (–5) + [18 + (–8)] = [–5 + 18] + (–8)?

(*iii*) What is –75 + 0 equal to? Also state the property.

(*iv*) Write the following integers in ascending order : 6 , 0, –2, 1, 5, –4

10. (*i*) Write a negative integer and a positive integer whose difference is –4.

(*ii*) Use distributive property of multiplication over addition, find the value of 12 × 15.

(*iii*) Write two integers which is smaller than –7 but their difference is –7.

(*iv*) An elevator descends into a mine shaft at the rate of 8 m/min. If the descent starts from 20 m above ground level, how long will it take to reach –460 m.

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